Topology-based Feature Extraction from 3D Scalar Fields

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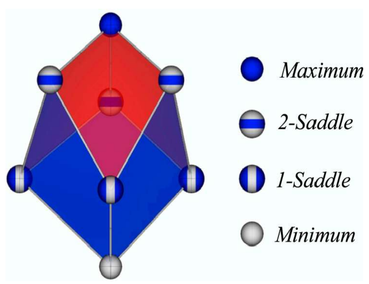
In this project, we investigate a topological approach for simplifying continuous functions defined on volumetric domains. We introduce two atomic operations that remove pairs of critical points of the function and design a combinatorial algorithm that simplifies the Morse-Smale complex by repeated application of these operations. The Morse-Smale complex is a compact representation of gradient flow between critical points of a function, and critical points paired by the complex represent features and their importance. By manipulating the complex, we are able to modify features directly, enabling controlled simplification of the function.

Motivation

- **The Morse-Smale complex is an efficient representation of data through gradient flow.**
- **Topology-based simplification reveals features in a controlled manner.**
- **Other uses such as noise removal, smoothing, and automatic transfer function generation.**

Morse-Smale complex

The Morse-Smale complex is a segmentation of the domain into regions of uniform gradient flow behavior. These regions are the intersections of descending manifolds and ascending manifolds, and are called crystals. The boundary of a crystal consists of lower-dimensional cells: quads, arcs, and nodes.



Every crystal has a unique origin and destination node, the minimum and maximum, respectively, which are end points of integral lines lying within.

Figure: *A typical crystal of the Morse-Smale complex.*

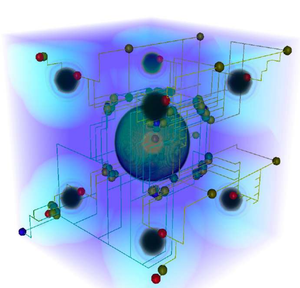


Figure: *The MS complex of a function consisting of various spikes with the central one being the largest. Maxima are rendered as red spheres, minima as blue, and 1- and 2-saddles as yellow and teal respectively.*

Simplification

We simplify the function through the atomic operations of critical point pair cancellation. A cancellation simulates the smoothing of the function by modifying the gradient flow in the neighborhood of two critical points. The critical points that are cancelled must be connected by arcs in the complex, and the cancellation must result in a valid complex and must not alter the topology of the domain. We order cancellations in order of increasing persistence, the absolute difference in function value of a pair of critical points.

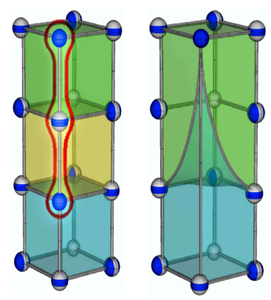


Figure: *A saddle-maximum cancellation simulates the merging of three critical points, which corresponds to the removal of a bump from function. Notice that the yellow crystal was removed.*

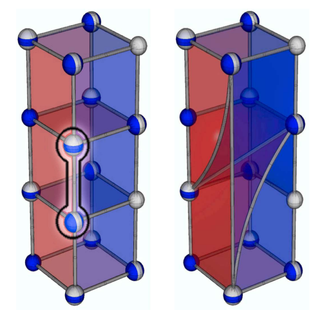


Figure: *A 1-saddle 2-saddle cancellation smooths the function, but increases the complexity of the complex. New cells are introduced that separate crystals incident on the arc that is removed.*

Results

We pre-process the data to compute a Morse-Smale complex. Subsequent cancellations remove noise and reveal features, which we highlight using an automatically generated transfer function. Exploration and simplification of the data are performed interactively. The combinatorial nature of our algorithm leads to a robust and efficient implementation. Current goals are to extend our implementation to handle large scale data, and to develop a numerical realization of the simplification procedure.

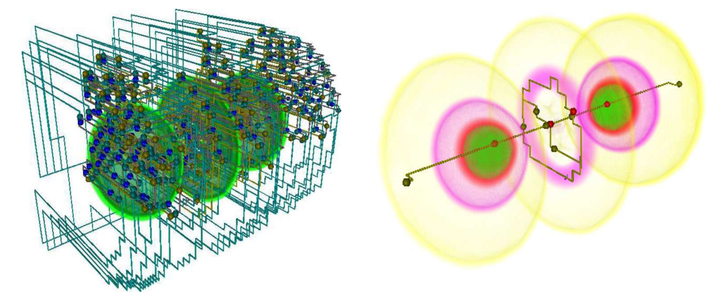


Figure: *The initial Morse-Smale complex of spatial probability distribution of electrons in a hydrogen atom under a large magnetic field (left). The complex reveals the features after simplification (right).*

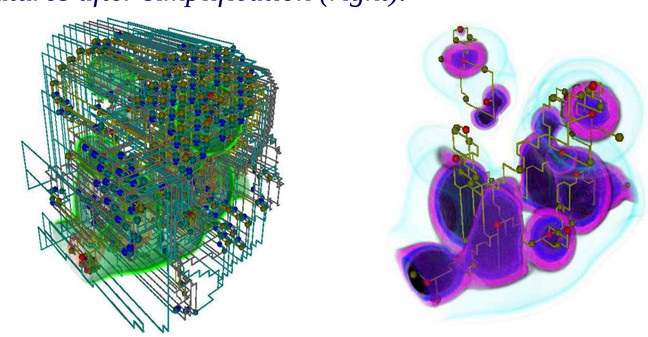


Figure: *Features identified from the neghip dataset reveal the various clusters of atoms present in the protein.*

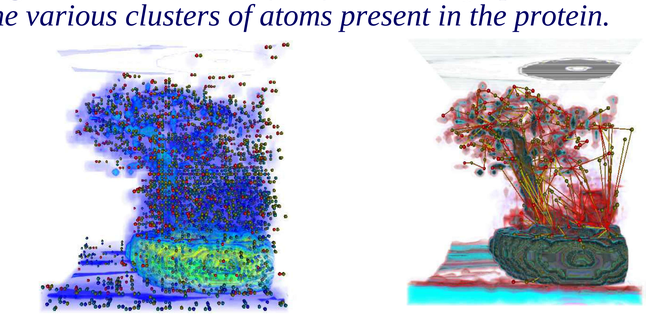


Figure: *Arcs in the simplified complex trace out the branching structure of a CT scan of a bonsai tree.*

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